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Sharp-Edged Rectangular Wing Characteristics

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Nomenclature

A = aspect ratio
c = wing chord
c.p. = center of pressure
c.g. = center of gravity, reference point, 0.25c
ΔC_D $= C_D - C_{D_0} = \dfrac{\text{lift dependent drag}}{qS}$, lift dependent drag coefficient
C_L $= \dfrac{\text{lift}}{qS}$, lift coefficient
C_m $= \dfrac{\text{pitching moment}}{qSc}$, pitching moment coefficient
C_N $= \dfrac{\text{normal force}}{qS}$, normal force coefficient
K_p $= \dfrac{\partial (C_{N,p})}{\partial (\sin\alpha\cos\alpha)}$
$K_{v,\mathrm{le}}$ $= \dfrac{\partial(2 \text{ leading edge suction force from one side}/qS)}{\partial \sin^2\alpha}$
$K_{v,\mathrm{se}}$ $= \dfrac{\partial(2 \text{ tip suction force from one side edge}/qS)}{\partial \sin^2\alpha}$
M = Mach number
q = freestream dynamic pressure
S = wing area
x = streamwise coordinate; origin at the leading edge
α = angle of attack, deg
β $= \sqrt{1-M^2}$
$\Delta\xi$ $= \dfrac{x_{\mathrm{ref}} - x_i}{c}$, dimensionless distance between reference point and c.p. of the aerodynamic item in question

Subscripts
v,le = vortex effect at the leading edge
p = potential or attached flow
v,se = vortex effect at the side edge

Introduction

SINCE the early days of applied aerodynamics (Prandtl's lifting line solution) the estimation of at least the symmetric characteristics of the rectangular wing has been a challenge to the aerodynamicist.

After the lifting line concept, valid for straight wings of large aspect ratio, numerical methods were developed within the lifting surface concept during the 1930s and 1940s, giving more freedom as to the choice of planform. During the 1960s the so-called panel methods were introduced and the planform flexibility was still further improved. All these numerical methods are usable within the potential flow regime (linear theory). Since about 1975 promising work has appeared where the panelization is extended to include even the vortex sheets from the edges of wings with separated flow so that non-conical effects could be estimated. When methods of this type are completely developed it will be possible to obtain the nonlinear characteristics of wings with edge-separated flow in a straight-forward numerical computation. However, it must be recognized that an appreciable amount of computer time will be required to obtain results over an angle of attack range.

As to the rectangular wing with sharp edges, the subject of the present Note, the most practical numerical method from an engineering point of view up to now has been developed at NASA in the form of a panelized Multhopp method[1] in combination with the suction analogy.[2] The numerical results correlate very well with experiments as long as the separated flow reattaches on the wing surface and any shock waves which may appear on the wing are still weak.

By an inspection of the above numerical results it has now been possible to represent them (semiempirically) by analytical expressions of great simplicity partly known from earlier theoretical contributions.

Analytical Formulas

In the nomenclature of Refs. 1 and 2 the lift, induced drag, and pitching moment coefficients are, respectively,

$$C_L = K_p \sin\alpha\cos^2\alpha + (K_{v,\mathrm{le}} + K_{v,\mathrm{se}})\sin\alpha\,|\sin\alpha|\cos\alpha \tag{1}$$

$$\Delta C_D = C_L \tan\alpha \tag{2}$$

$$C_m = K_p \sin\alpha\cos\alpha\Delta\xi_p + K_{v,\mathrm{se}}(K_{v,\mathrm{le}}\Delta\xi_{\mathrm{le}}\Delta\xi_{\mathrm{se}})\sin\alpha\,|\sin\alpha| \tag{3}$$

The K coefficients have been determined numerically by Lamar[1] as functions of βA. However, the c.p. values for the respective contribution to the pitching moment were not explicitly given. The reference point for the pitching moment is 25% of the chord.

In Fig. 1 Lamar's[1] K coefficients are compared with the following analytical representations.

$$\beta K_p = 2\pi\beta A / [2 + \sqrt{4/3(\beta A)^2 + 4}] \tag{4}$$

$$\beta K_{v,\mathrm{le}} = \pi\beta A / [2 + \sqrt{1/4(\beta A)^2 + 4}] \tag{5}$$

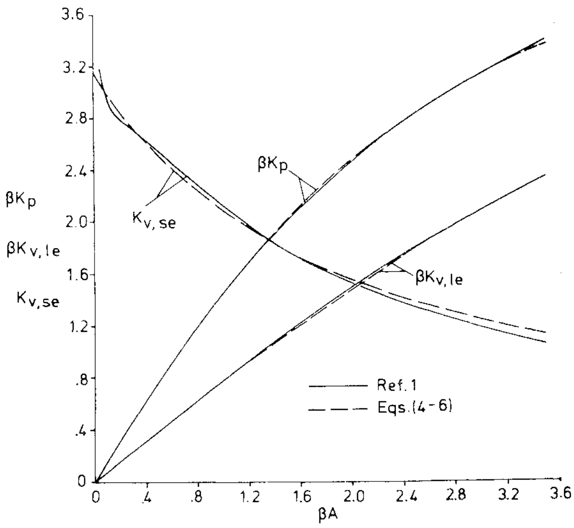

Fig. 1 Variation of βK_p, $\beta K_{v,\mathrm{le}}$, and $K_{v,\mathrm{se}}$ with aspect ratio and Mach number for rectangular wings according to Lamar, Ref. 1, compared with the semiempirical expressions, Eqs. (4-7).

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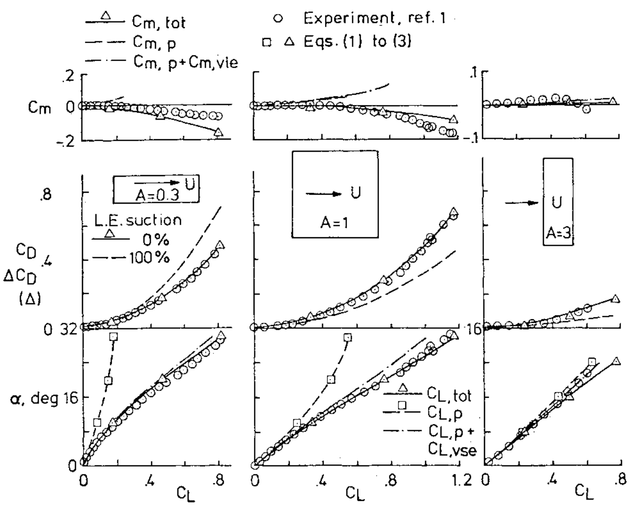

Fig. 2 Lift, drag, and pitching moment coefficients for sharp edged rectangular wings with aspect ratios $A=0.3$, 1, and 3 at $M=0.2$. Numerical solution and experiment by Lamar, Ref. 1, compared with the semiempirical expressions, Eqs. (1-3).

$$K_{v,\mathrm{se}} = 2\pi/(2+\beta A) \tag{6}$$

K_p is equivalent to the coefficient C_{L_α} in potential flow. If in Eq. (4) the factors β and $4/3\beta^2$ are put to unity, what remains is recognized as Helmbold's[3] result, obtained by a modification of the lifting line idealization. Helmbold's result differs from the simple lifting line result, which is incorrect for $A<5$, approximately.

Equation (5) gives the suction force coefficient at the leading edge. The formal resemblence between K_p and $K_{v,\mathrm{le}}$ is perhaps not too surprising.

Equation (6) expresses the suction force coefficient at the side edges. If β is set equal to one, the factor multiplying the aspect ratio, A, in Prandtl's lifting line result, $C_{L_\alpha}=2\pi A/(2+A)$, is recognized. Therefore the result of the simple lifting line theory is related to the suction force coefficient at the side edges.

The constants 4/3 and 1/4 in Eqs. (4) and (5) are obtained by curve-fitting and bring the corresponding curves to correlate mostly within plotting accuracy. The constants could in fact be looked upon as akin to profile efficiency factors. Equation (6) correlates with Lamar's[1] result for $A<2$ to within a 2% deviation. For larger aspect ratios the discrepancy increases, but is still less than 6% at $A=3$. The deviation could of course be decreased if desired, but not eliminated completely using only one adjustable constant.

In Eq. (3) the $\Delta\xi$ terms represent the dimensionless distance between the reference point ($0.25c$) and the c.p. values of the respective, indexed, aerodynamic items. Here the simplest possible assumptions are made about the dimensionless c.p. locations, namely,

$$\Delta\xi_p = 0, \qquad \Delta\xi_{\mathrm{le}} = 1/4, \qquad \Delta\xi_{\mathrm{se}} = -1/4 \tag{7}$$

Results and Discussion

Figure 2 shows Lamar's[1] numerical results compared with experiments at $M=0.2$ on sharp edged rectangular wings with aspect ratios $A=0.3$, 1, and 3. Substituting Eqs. (4-7) into Eqs. (1-3) and evaluating as a check at some angles of attack for the three wings gives the □-symbol points in Fig. 2 for attached (linear, potential) flow and the △-symbol points for the total coefficients.

Noticing that C_{D_0} (the zero lift drag coefficient) has to be added to ΔC_D [the lift dependent drag coefficient, Eq. (2)] shown in the figures, it is evident that the correlation between the analytical formulas and Lamar's numerical, extended Multhopp method solution is very good, as it should be. If the potential flow pitching moment contribution were estimated with Eq. (3), and presented in Fig. 2, a discrepancy would be seen. This would be expected to occur because of the constant $x_p = 0.25c$ assumed here, since it is known that c.p. moves forwards with decreasing aspect ratio. The correlation between total pitching moments at higher angles of attack shows that the assumed three c.p. locations in Eq. (7) are, taken together, consistent, though all three of them might be incorrect individually. In a real flow they certainly are; that can be seen when the theoretical pitching moment is compared with the experiments as in Fig. 2.

Conclusion

The above findings make it legitimate to look upon the set of formulas, Eqs. (1-7), as a semiempirical solution of Lamar's[1] numerical, extended Multhopp method when applied to rectangular wings with sharp edges and to the symmetric characteristics of those wings.

Acknowledgment

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